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Contributions to Plasma Physics

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Journal:	*Contributions to Plasma Physics*
Manuscript ID:	ctpp.200900001.R1
Wiley - Manuscript type:	Symposia and Congresses
Date Submitted by the Author:	
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Keywords:	ballooning modes, LHD

Contributions to Plasma Physics, 9 October 2009

Drift stabilisation of ballooning modes in an inward-shifted LHD configuration

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Received XXXX, revised XXXX, accepted XXXX
Published online XXXX

Key words Drift magnetohydrodynamics, ballooning, diamagnetic, LHD.

PACS 52.35.Py,52.55.Hc

A drift-magnetohydrodynamic theory is applied to a background anisotropic pressure equilibrium state to generate a drift corrected ballooning mode equation. The ratio of the mode frequency to the hot particle drift frequency constitutes the critical expansion parameter. The fast particles thus contribute weakly to the instability driving mechanism and also to the diamagnetic drift stabilisation. This equation is used to model the inward-shifted Large Helical Device (LHD) configuration. In the single-fluid limit, a weakly ballooning unstable band that encompasses a third of the plasma volume develops in the core of the plasma at low $\langle\beta_{dia}\rangle$ that becomes displaced towards the edge of the plasma at the experimentally achieved $\langle\beta_{dia}\rangle \simeq 5\%$. Finite diamagnetic drifts (mainly due to the thermal ions) effectively stabilise these ballooning structures at all values of $\langle\beta_{dia}\rangle$. The validity of the large hot particle drift approximation is verified for hot to thermal ion density ratios that remain smaller than 2%.

1 Introduction

Ideal magnetohydrodynamic (MHD) theory yields inconclusive predictions with regards to the stability of the Large Helical Device (LHD) [1–3]. Experimental discharges have achieved $\langle\beta_{dia}\rangle \simeq 5\%$, where $\langle\beta_{dia}\rangle$ corresponds to the volume average of the diamagnetic component of β [4,5]. These conditions are generated with high power tangential neutral beam injection (beam particle energy $\simeq 180kev$, beam power $\simeq 14MW$) at relatively low densities ($< 3 \times 10^{-19} m^{-3}$) [4]. At these low densities, the beam driven pressure anisotropy can be quite significant [6].

Ideal ballooning modes in three-dimensional (3D) systems can impose severe restrictions on the total pressure that can be confined [7, 8], in particular on those field line that traverse the most destabilising curvature region. The combination of finite thermal diamagnetic drifts and plasma compression is speculated to be adequate to guarantee stability at high $\langle\beta_{dia}\rangle$ in the LHD device, but would be insufficient at lower $\langle\beta_{dia}\rangle$ [3].

We consider the application of a drift-corrected ballooning mode equation valid for anisotropic pressure background equilibria in which the anisotropy is driven by the energetic beam particles [9]. The 3D anisotropic pressure equilibria required to model the inward-shifted LHD configuration are computed with the ANIMEC code [10] The fast particle distribution function is approximated with a special bi-Maxwellian that satisfies the lowest order solution of the Fokker-Planck equation [11].

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2 The drift-stabilised ballooning mode equation

A drift-corrected ballooning mode equation [9] is obtained from the linearised gyrokinetic equation in ballooning space [12] which has been expanded in the limit that the mode frequency ω is much smaller than the hot particle curvature $+\nabla B$ drifts (ω_{dh}). In variational form, it can be written as

$$\begin{aligned} &\int_{-\infty}^{\infty} d\theta \Bigg\{ \frac{\sigma k_\perp^2}{\sqrt{g} B^2} \left| \sqrt{g} \boldsymbol{B} \cdot \boldsymbol{\nabla} \chi \right|^2 - \frac{k_\alpha \sqrt{g} p'(s)}{\psi'(s) B^2} \cdot \\ &\left[\left(1 + \frac{\sigma}{\tau} \right) \boldsymbol{B} \times \boldsymbol{k}_\perp \cdot \boldsymbol{\kappa} - \frac{k_\alpha}{\psi'(s)} \frac{\partial p_{\perp h}}{\partial s} \bigg|_B \right] |\chi|^2 \Bigg\} \\ = \; &\int_{-\infty}^{\infty} d\theta M_i N_i \frac{\sqrt{g} k_\perp^2}{B^2} \omega \left[\omega - \left(\omega_{*pi} + \frac{M_h N_h \omega_{*h}}{M_i N_i b_h} \right) \right] |\chi|^2 \end{aligned} \tag{1}$$

where the first term on the left handside corresponds to the field line bending stabilisation, the second term constitutes the instability drive due to the interaction of the pressure gradients mainly due to the thermal component p and the magnetic field line curvature. The right hand side corresponds to the plasma inertia. In the limit $\omega_{*pi} \to 0$ and $\omega_{*h} \to 0$, the mode frequency $\omega \to \gamma_F$, where γ_F constitutes the fluid growth rate. The Jacobian is denoted by $\sqrt{g}$, θ is the poloidal angle, the wave vector is $\mathbf{k}_\perp$, κ is the magnetic field line curvature, s is the radial variable proportional to the enclosed toroidal magnetic flux, $\mathbf{B}$ is the magnetic field, M_j and N_j denote the particle mass and density of ionic species j, respectively, ω_{*pi} is the total ion diamagnetic drift frequency, ω_{*h} is the hot particle diamagnetic drift frequency, $b_h = k_\perp^2 \rho_h^2$, and ρ_h is the hot particle Larmor radius. The ballooning eigenfunction is given by χ. The ballooning equation Eqn. 1 is solved perturbatively in two steps. We first compute the single fluid ballooning eigenvalue $\lambda = -\gamma_F^2$ with the BECOOL code [13]. Using the χ obtained as a test function, we derive the quadratic dispersion relation

$$\omega^2 - \omega(\omega_{*pi} + \omega_{*heff}) + \gamma_F^2 = 0 \tag{2}$$

where ω_{*heff} is the effective hot particle diamagnetic drift frequency:

$$\omega_{*heff} = \left\langle \frac{M_h N_h \omega_{*h}}{M_i N_i b_h} \right\rangle = \frac{\int_{-\infty}^{\infty} d\theta \frac{\sqrt{g}}{\psi'(s)} \left(\frac{k_\perp^2}{B^2} \right) \left(\frac{M_h N_h \omega_{*h}}{M_i N_i b_h} \right) |\chi|^2}{\int_{-\infty}^{\infty} d\theta \frac{\sqrt{g}}{\psi'(s)} \left(\frac{k_\perp^2}{B^2} \right) |\chi|^2} \tag{3}$$

Frequencies and growth rates are normalised to the toroidal Alfvén frequency $\omega_A = v_A/R_0$ where $v_A = B_0/\sqrt{\mu_0 M_i N_{io}}$, R_0 is the major radius and the subscript 0 denotes an evaluation at the magnetic axis.

3 Computational modelling

For simplicity, we restrict the studies to a single field line that traverses the outside edge of the horizontally up-down symmetric cross section in the LHD device as this point aligns with the position where the magnetic field line curvature on each magnetic flux surface is most destabilising [1].

In the single fluid limit ($\omega_{*pi} = \omega_{*heff} = 0$), ballooning modes are unstable at all values of $\langle\beta_{dia}\rangle$ [14]. The ballooning unstable band exists in the central core of the plasma $0 < s < 0.36$ and moves radially outward with increasing $\langle\beta_{dia}\rangle$. At $\langle\beta_{dia}\rangle = 2.5\%$, the unstable domain lies at $0.5 < s < 0.8$ while at $\langle\beta_{dia}\rangle = 4.8\%$, it radicates at $0.6 < s < 0.9$.

The solution of the dispersion relation Eqn. 2 reveals that the ballooning modes become stable for all values of $\langle\beta_{dia}\rangle$ in the range $0 < \langle\beta_{dia}\rangle < 4.8\%$ that we have investigated because we obtain that $|\omega_{*pi} + \omega_{*heff}|/2 >> \gamma_F$. Therefore, the mode frequency is real. Under conditions that $|\omega_{*pi} + \omega_{*heff}|/2 < \gamma_F$ the frequency is complex, with its imaginary component corresponding to the growth rate.

We have therefore established the stability properties of the drift-magnetohydrodynamic model we have considered to the inward-shifted LHD configuration. We must now address the limits of validity of this model. For this purpose, it is appropriate to define a mode-width averaged hot particle drift frequency. The rationale behind this is that ballooning mode structures can become very localised along a magnetic field line and therefore the extent of the contribution of the energetic particle diamagnetic drifts to the stabilisation of the modes should be restricted to the domain where the instability structure has finite amplitude [14]. Consequently, we define the average

$$\langle \omega_{dh} \rangle = \frac{\int_{-\infty}^{\infty} d\theta(\sqrt{g}/\psi')\omega_{dh}|\chi|^2}{\int_{-\infty}^{\infty} d\theta(\sqrt{g}/\psi')|\chi|^2} \quad . \tag{4}$$

We plot the profiles of $\omega/\langle \omega_{dh} \rangle$ for the wave number $k_\alpha = 10$ at $\langle \beta_{dia} \rangle = 1\%$ in Fig. 1 and at $\langle \beta_{dia} \rangle = 4.5\%$ in Fig. 2 for ratios of energetic particle to thermal ion density between 1% and 5%. The on axis fast particle density is denoted by N_{h0} while that of the thermal ions is N_{i0}. It should be noted that for $\langle \beta_{dia} \rangle = 1\%$ the profiles are shown for $s < 0.55$ while those at $\langle \beta_{dia} \rangle$ are in the range $0.57 < s < 0.89$. These correspond to the domains for which γ_F is unstable. Outside the γ_F unstable bands, the eigenfunctions χ emerge from the Alfvén continuum (as $\gamma_F \geq 0$) and can have a significantly different structure than the ballooning modes with $\gamma_F < 0$. The computation of $\langle \omega_{dh} \rangle$ in the stable band can then become misleading. We see that for $\langle \beta_{dia} \rangle = 1\%$, the ratio $\omega/\langle \omega_{dh} \rangle$ remains well below unity even for $N_{h0}/N_{i0} = 5\%$, however, at $\langle \beta_{dia} \rangle = 4.5\%$ the validity of the small $\omega/\langle \omega_{dh} \rangle$ breaks down for $N_{h0}/N_{i0} > 2.5\%$. Further confirmation is provided in Fig. 3 where the ratio $\omega/\langle \omega_{dh} \rangle$

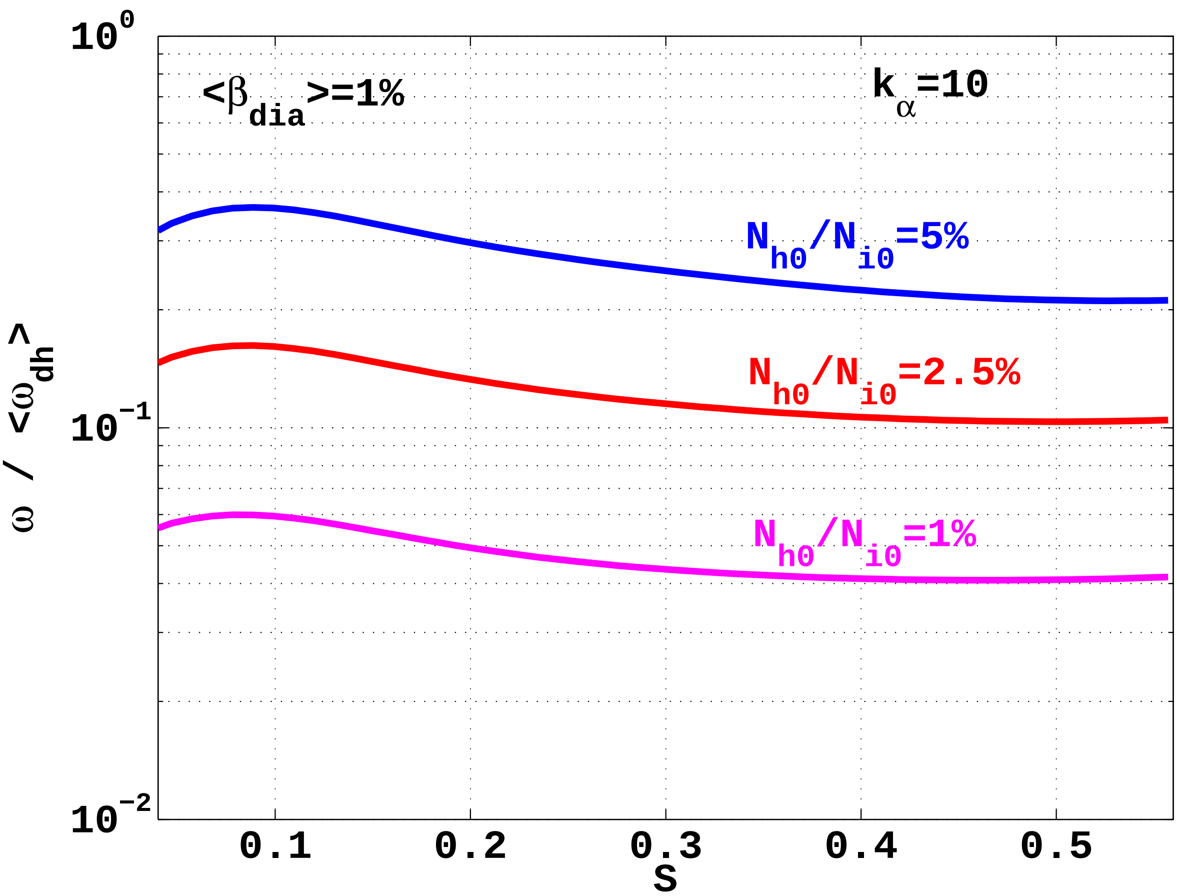

Fig. 1 The $\omega/\langle \omega_{dh} \rangle$ profiles at $\langle \beta_{dia} \rangle = 1\%$ and k_α for on-axis hot to thermal ion density ratios of $N_{h0}/N_{i0} = 1\%$, 2.5% and 5%.

is displayed as a function of N_{h0}/N_{i0} when $k_\alpha = 10$ for $\langle\beta_{dia}\rangle = 4.5\%$ and for $\langle\beta_{dia}\rangle = 1\%$ at radial positions $s = 0.6$ and $s = 0.1$, respectively.

The dependence of $\omega/\langle\omega_{dh}\rangle$ on k_α near the edge of the plasma (at $s = 0.87$) is shown for $\langle\beta_{dia}\rangle = 4.5\%$ considering three ratios of N_{h0}/N_{i0} in Fig. 4.

4 Summary and conclusions

We have applied a drift-corrected ballooning mode equation that can describe the impact of strongly drifting energetic particles, that drive a pressure anisotropy, to the LHD inward-shifted configuration where $\langle\beta_{dia}\rangle \simeq 5\%$ conditions have been attained.

The ballooning modes that we compute are unstable in the absence of diamagnetic drift corrections to the plasma inertia. These drifts, mainly due to the thermal ions, effectively stabilise the single fluid eigenstructures because the condition $|\omega_{*pi} + \omega_{*heff}| >> 2\gamma_F$ is satisfied for all values of $\langle\beta_{dia}\rangle$. The mode-width averaged fast particle diamagnetic drifts can become dominant for low wave number k_α where the applicability of the ballooning approximation is questionable. At low $\langle\beta_{dia}\rangle \sim 1\%$, the hot particle to thermal ion density ratio can exceed 5% and still remain within the bounds of validity of the small ω/ω_{dh} expansion. This ratio decreases to $\sim 2\%$ at higher $\langle\beta_{dia}\rangle \sim 5\%$.

The drift magnetohydrodynamic theory that we have applied to the LHD configuration examined may be a more appropriate model to describe the experimental observations.

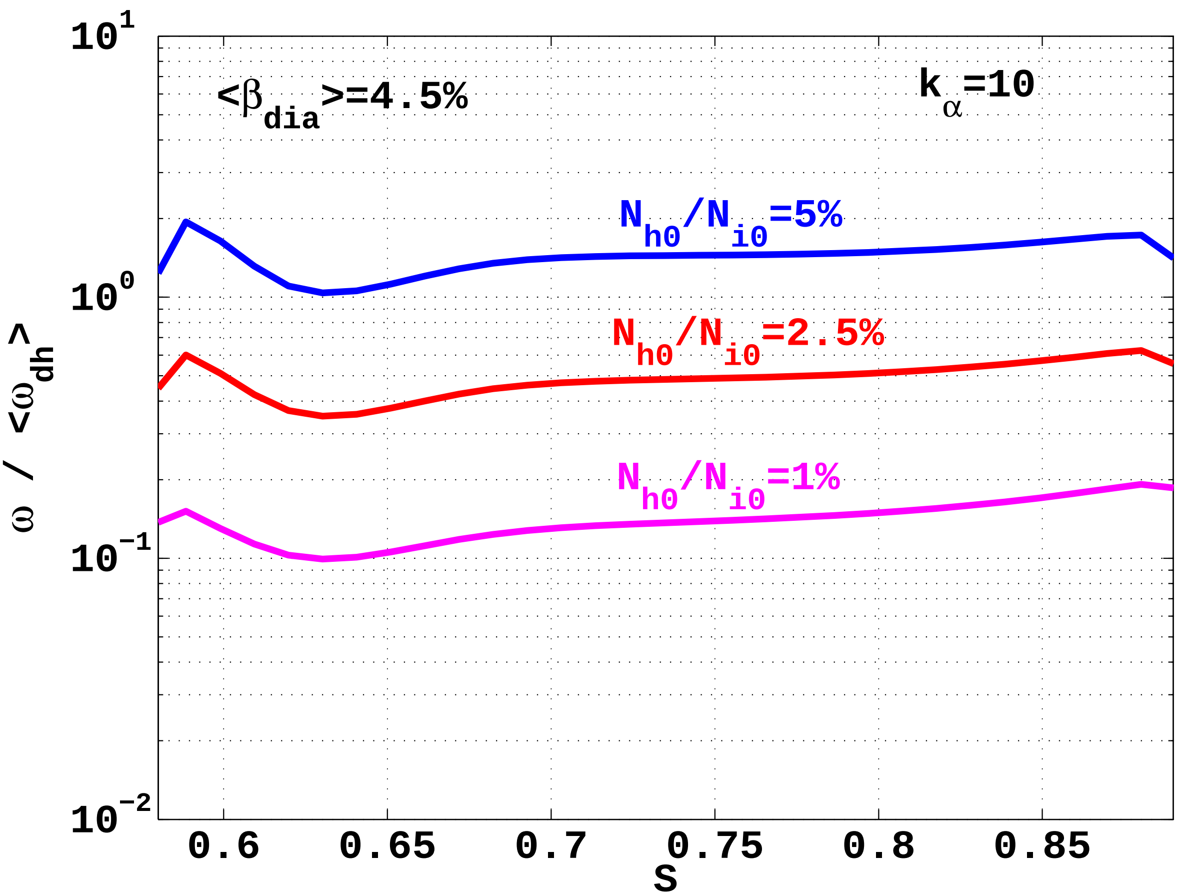

Fig. 2 The $\omega/\langle\omega_{dh}\rangle$ profiles at $\langle\beta_{dia}\rangle = 4.5\%$ and k_α for on-axis hot to thermal ion density ratios of $N_{h0}/N_{i0} = 1\%$, 2.5% and 5%.

Acknowledgements This work was supported in part by the Swiss National Science Foundation and by Euratom.

References

[1] N. Nakajima, Phys. Plasmas **3**, 4556 (1996).
[2] N. Nakajima, S. R. Hudson, and C. C. Hegna, Fusion Science and Technol. **51**, 79 (2007).
[3] N. Nakajima, S. R. Hudson, C. C. Hegna, and Y. Nakamura, Nucl. Fusion **46**, 177 (2006).
[4] K. Y. Watanabe, S. Sakakibara, Y. Narushima et al., Nucl. Fusion **45**, 1247 (2005).
[5] A. Weller, K. Y. Watanabe, S. Sakakibara et al., Nucl. Fusion **49**, 065016-1 (2009).
[6] T. Yamaguchi, K. Y. Watanabe, S. Sakakibara et al., Nucl. Fusion **45**, 133 (2005).
[7] D. Correa-Restrepo, Z. Naturforsch. **33a**, 789 (1978).
[8] R. L. Dewar and A. H. Glasser, Phys. Fluids **26**, 3038 (1983).
[9] W. A. Cooper, Phys. Fluids **26**, 1830 (1983).
[10] W. A. Cooper, S. P. Hirshman, P. Merkel, J. P. Graves, J. Kisslinger, H. F. G. Wobig, Y. Narushima, S. Okamura, and K. Y. Watanabe, Comp. Phys. Commun. **180**, 1524 (2009).
[11] K. G. McClements, R. O. Dendy, R. J. Hastie, and T .J. Martin, Phys. Plasmas **3**, 2994 (1996).
[12] P J. Catto, W M. Tang, and D. E. Baldwin, Plasma Phys. **23** 639 (1981).
[13] G. A. Cooper, J. P. Graves, W. A. Cooper, R. Gruber, and R. S. Peterson, J. Comput. Phys. **228** 4911 (2009).
[14] L. Brocher, W. A. Cooper, J. P. Graves, G. A. Cooper, Y. Narushima, and K. Y. Watanabe, submitted to Nucl. Fusion (2009).

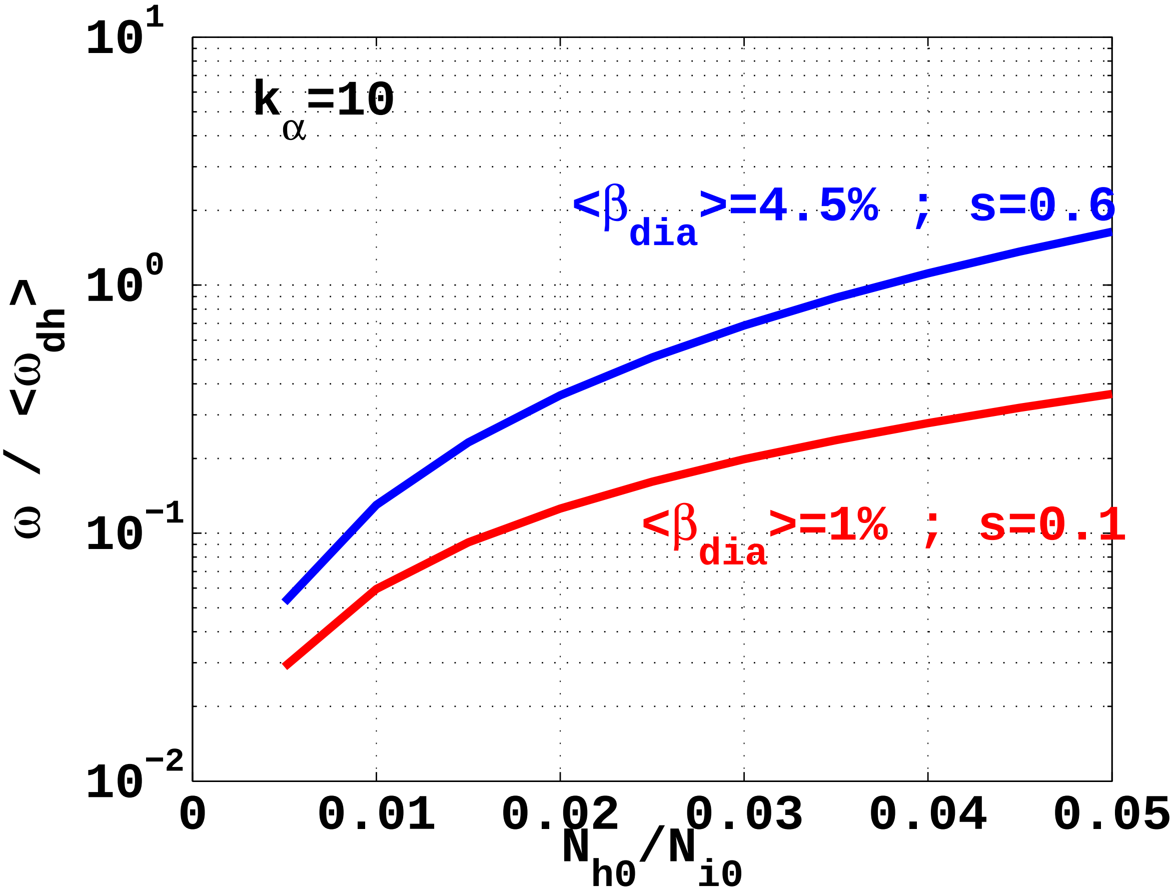

Fig. 3 $\omega/\langle\omega_{dh}\rangle$ as a function of N_{h0}/N_{i0} with $k_\alpha = 10$ for $\langle\beta_{dia}\rangle = 4.5\%$ at $s = 0.6$ and $\langle\beta_{dia}\rangle = 1\%$ at $s = 0.1$.

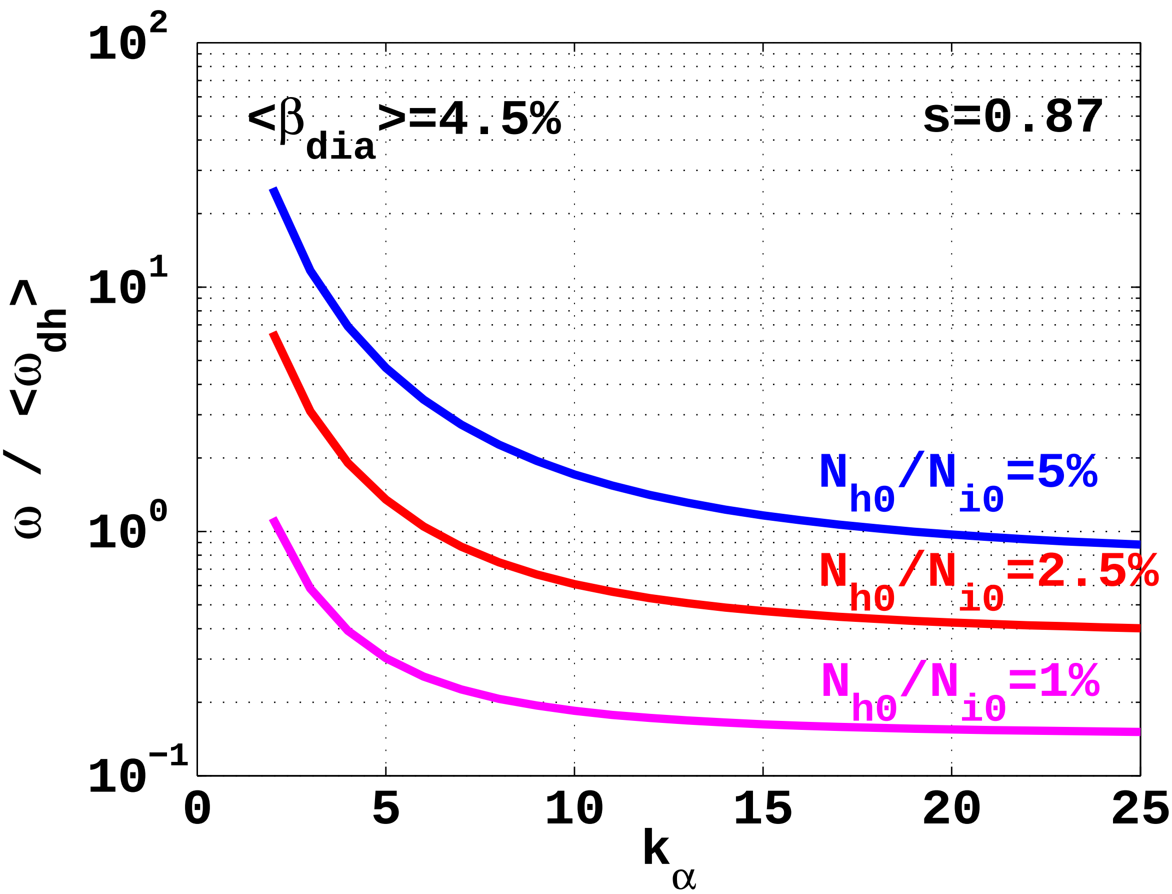

Fig. 4 $\omega/\langle\omega_{dh}\rangle$ as a function of k_α at $\langle\beta_{dia}\rangle = 4.5\%$ on the flux surafe with $s = 0.87$ for on-axis hot to thermal ion density ratios of $N_{h0}/N_{i0} = 1\%$, 2.5% and 5%.